Extra Problems #2

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1: Kronk and Yzma are high school students are applying for Stanford, and they each take standardized tests. Kronk takes the ACT and scores a 23 (the mean for the ACT is 20.8 and the standard deviation is 4.8). Ezma takes the SAT and scores 1600 (the mean for the SAT is 1500 and the standard deviation is 250).
- a) What percentage of students did Kronk score higher than?
- b) What percentage of students did Yzma score lower than?
- c) Another high school student, Kuzco, takes the SAT and scores higher than exactly 90% of students. What was his score?

2: Simba and Pumbaa schedule a meeting at 11:30. Pumbaa arrives sometime between 11:25 and 11:40 (with an equal chance of arriving at any time within that window).
- a) If Simba arrives exactly at 11:30, how likely is he to arrive before Pumbaa?
- b) If Simba arrives between 11:30 and 11:45 (again, with an equal chance of arriving any time in that window), how likely is he to arrive before Pumbaa?

3: Aladdin and Jasmine heard that there is a magic lamp in a street in Agrabah, and are searching for it. The street is 100 meters long, and the probability they find a magic lamp at any point is:

$$f(x) = \begin{cases} c(100 - x^2) & if \ 0 > x > 100 \\ 0 & otherwise \end{cases}$$

- a) What is c?
- b) What is the probability they find the lamp in the first x meteres?
- c) What is the probability they find the lamp in the first 50 meters?
- d) If they start at the beginning of the street and walk towards the end of the street, when do you expect them to find the lamp?
- e) How much more likely is it that they find the lamp at 50 meters vs 75 meters?

4: Mowgli is foraging for fruit in a forest. He can either go to a grove of orange trees, or a grove of mango trees. The number of mangos he would collect can be modeled by a Poisson distribution with lambda = 4, and the number of oranges he would collect can be modeled as a Poisson distribution with lambda = 15. Each mango has 200 calories, and each orange has 50 calories.

a) Which grove should he go to in order to maximize the probability of getting at least 450 calories?

b) Last year, Mowgli went to the mango grove 219 days (roughly 60% of a year) and went to the orange grove the remaining days. Suppose one day Mowgli walked back with 600 calories. What's the probability he went to the orange grove?

5: Robin Hood and Maid Marian are playing a game where they roll dice until they both roll the same number.

    a) What's the probability they roll the dice less than or equal to five times?

    b) How many rolls do you expect it will take them to both roll the same number?

6: You're taking the CS 109 final, and you reach a multiple choice section. There are 10 questions, each with four possible answers. You know you need to get at least 7 of those questions correct pass the class. If you were to guess randomly on each question, what's the probability you will pass the class?